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PSEUDOSCALAR NUCLEON FORM FACTOR FROM INVERSE PION ELECTROPRODUCTION AND THE FIRST RADIAL PION EXCITATION

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The remarkable property of minimization of rescattering effects in the inverse pion electroproduction (IPE) at low energies at the «quasi-threshold» allows one to justify a method of determining the weak structure (in addition to the electromagnetic one) of a nucleon from experimental data on this process by using the current algebra description. The result of extracting the pseudoscalar nucleon form factor $G_p(t)$ by that method from the IPE data in the first πN resonance region is presented, where the definite indication of the existence of the state π' in the range 500–800 MeV (possibly the first radial excitation of pion) is obtained.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Псевдоскалярный формфактор нуклона из обратного электророждения пионов и первое радиальное возбуждение пиона

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Замечательное свойство минимизации эффектов перерассеяния в обратном электророждении пионов (IPE) при низких энергиях на «квазипороге» позволяет обосновать метод определения слабой структуры (в дополнение к электромагнитной) нуклона из экспериментальных данных по этому процессу с использованием описания на основе алгебры токов. Представлен результат выделения таким методом из IPE-данных в первой πN-резонансной области псевдоскалярного формфактора нуклона $G_p(t)$, где получено определенное указание на существование состояния π' в интервале 500—800 МэВ (возможно, первого радиального возбуждения пиона).

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1. The process $\pi N \to e^+e^-N$ (inverse pion electroproduction — IPE), being a natural and unique laboratory for studying the hadron electromagnetic structure in the timelike region of the virtual-photon «mass» λ^2, turns out to be also very useful for investigating the weak structure of a nucleon. The latter investigation is based on the current algebra (CA) description and the remarkable property of IPE according to which the e^+e^- pairs of maximal masses (at the «quasi-threshold») are created by the Born mechanism with the rescattering-effect contributions at the level of radiative corrections up to the total πN

energy $w \approx 1500$ MeV (the «quasi-threshold theorem») [1]. Therefore, the threshold CA theorems for pion electro- and photoproduction can be justified in the case of IPE up to the indicated energy [2,3]. On the one hand, this allows one to avoid threshold difficulties when using IPE (unlike electroproduction) for extracting weak form factors (FF's) of a nucleon. On the other hand, there is no strong kinematical restriction inherent in μ capture and is no kinematical suppression of contributions of the induced pseudoscalar nucleon FF to cross sections of «straight» processes as $\nu N \to lN$ present in due to multiplying by lepton masses.

2. The research of pion photo-, electroproduction and IPE in one-photon approximation is related with studying the amplitude of the process $\gamma^* N \rightleftarrows \pi N'$, $J_\mu(s, t, \lambda^2)$, where $s = (p_1 + q)^2$, $t = (k - q)^2$ are usual Mandelstam variables and $\lambda^2 = 0, < 0$ and $4m_e^2 \le \lambda^2 \le \le (\sqrt{s} - m)^2$ correspond to the above three processes, respectively (p_1, p_2, q, and k are 4-momenta of nucleons, of pion and of photon).

For obtaining a reliable information on the nucleon structure, it is important to find kinematical conditions, where the IPE dynamics is determined mainly by a model-independent part of interactions, the Born one. To this end, we shall use such general principles, as analyticity, unitarity and Lorentz invariance, and phenomenology of processes $eN \to e\pi^\pm N$, $\gamma N \to \pi^\pm N$, considered in the framework of the unified (including IPE) model.

Earlier it was shown that the model, based on the fixed-t dispersion relations without subtractions at a finite energy for isovector amplitudes with the spectral functions describing the magnetic excitation of the $P_{33}(1232)$ resonance and with the isoscalar amplitudes being the Born ones [4], is successful in the unified explanation of the experimental data on pion electro-, photoproduction and IPE in the total-energy region from the threshold up to $w \approx 1500$ MeV [5].

Application of this model to the calculations for IPE shows the interesting growth of the relative contribution of the Born terms with λ^2 [5]. This approximate dominance of the Born terms has a model-independent explanation and is related with the quasi-threshold theorem [1] which means that at the quasi-threshold ($|\mathbf{k}| \to 0$, $\lambda^2 \to \lambda^2_{\max} = (\sqrt{s} - m)^2$) the IPE amplitude becomes the Born one in the energy region from the threshold up to ~ 1500 MeV. That remarkable dynamics of IPE, distinguishing it essentually from photo- and electroproduction, is related to the fact that at the quasi-threshold only the electric (E_{0+} and E_{2-}) and longitudinal (L_{0+} and L_{2-}) dipoles survive owing to the $k \to 0$ behaviour and, therefore, the selection rules appear (from parity conservation and from that the stopped virtual photon has the angular momentum $J = 1$): at the quasi-threshold only the resonances with $J^P = \frac{1^-}{2}$ ($S_{11}(1535)$, $S_{31}(1650)$, $S_{11}(1700)$, etc.) and $J^P = \frac{3^-}{2}$ ($D_{13}(1520)$, $D_{33}(1670)$, etc.) survive in the s channel of the IPE. Furthermore, indeed, in this kinematic configuration the process is stipulated only by two independent dipole transitions (either electric or longitudinal), because from the causality (analyticity) the quasi-threshold constraints arise:

$$E_{0+} = L_{0+}, \qquad E_{2-} = -L_{2-}.$$

Since the s- and d-wave πN resonances are excited above 1500 MeV, one can expect that dipoles E_{0+} and E_{2-} are mainly the Born ones below this energy. All the multipole analyses of charged pion photoproduction agree with this; and, e.g., the dispersion-relation calculation has confirmed this fact at $\lambda^2 \neq 0$. Therefore, with a good accuracy (< 5%) we can write for the quasi-threshold IPE below ~ 1500 MeV:

$$\lim_{k \to 0} \frac{q}{k} \frac{d^2\sigma}{d\lambda^2 d\cos\theta} \approx \frac{\alpha}{12\pi} \frac{m^2}{(\sqrt{s}-m)^2} \frac{1}{s} \times$$
$$\times \left\{ (1+\cos^2\theta) \, | E_{0+}^{Born} + E_{2-}^{Born} |^2 + \sin^2\theta \, | E_{0+}^{Born} - 2E_{2-}^{Born} |^2 \right\}. \qquad (1)$$

Here θ is an angle between the momenta of the final nucleon and of the electron in the e^+e^- c.m. system.

On using the quasi-threshold theorem, the realistic (dispersion) model and the so-called «compensation curves» [6] (these are curves in the (s, t) plane along which the differential cross section is the Born one and which are constructed on the basis of comparison of photoproduction experimental data with the Born cross section using the existence theorem for implicit functions) for choosing the optimal geometry of experiment, the method of determining electromagnetic FF's from low energy IPE is based. This method has successfully been realized in experiments on nucleon and on nuclei ^{12}C and ^{7}Li [7], where first a number of FF values was obtained in the timelike λ^2 region from 0.05 to 0.22 $(GeV/c)^2$. The obtained F_1^v values are quite consistent with the calculations in the framework of the unitary and analytic vector-meson dominance model of the nucleon electromagnetic structure [8]. In the Table, the values of electromagnetic FF's, obtained in experiments on nucleons, we need later, are presented. Note that here the same experimental errors are cited for F_1^v and F^π because in this λ^2 range these FF's can be considered to be connected with each other by the relation $F_1^v(\lambda^2) - F^\pi(\lambda^2) = \Delta(\lambda^2)$. The quantity $\Delta(\lambda^2)$ has been taken from the dispersion calculations [9], and its theoretical uncertainty is significantly less than the one in the calculations of F_1^v and F^π in view of the compensation of a number of contributions to the spectral functions and due to the dominating influence of the contribution of the one-nucleon exchange in this quantity in the region $4m_\pi^2 \leq \lambda^2 \lesssim 20m_\pi^2$.

Table

λ^2, m_π^2	2.77	2.98	3.44	3.75	4.00	4.47	4.52	5.28	5.75	6.11
$F_1^v(\lambda^2)$	0.96	0.93	1.16	1.04	1.14	1.22	1.13	1.20	1.32	1.36
$F^\pi(\lambda^2)$	0.91	0.85	1.04	0.91	0.99	1.04	0.95	1.01	1.12	1.16
Error	0.10	0.09	0.10	0.08	0.16	0.10	0.09	0.09	0.10	0.08

3. Now, let us indicate another interesting possibility of investigating the weak nucleon structure related to the nucleon Gamov–Teller transition described by the matrix element

$$\langle N(p_2)\,|A^{\alpha}_{\mu}|\,N(p_1)\rangle = \bar{u}(p_2)\frac{\tau^{\alpha}}{2}\,[\gamma_{\mu}G_A(k^2) + k_{\mu}G_P(k^2)]\gamma_5 u(p_1), \tag{2}$$

where A^{α}_{μ} is the axial-vector current, $G_A(k^2)$ and $G_P(k^2)$ are the axial and induced pseudoscalar FF's.

An alternative description of IPE in the framework of the current commutators, PCAC and completeness allows one to derive a low-energy theorem at the threshold ($\mathbf{q}=0$, $\lambda^2 \to m_{\pi}^2$) related to approximate chiral symmetry and $O(m_{\pi}^2)$ corrections; and the quasi-threshold minimization of the continuum contribution makes it possible to justify this approach up to $w \approx 1500$ MeV [2] with the continuum corrections being practically the same as in the dispersion-relation description. Then, at the quasi-threshold, retaining only the leading terms in λ^2/m^2, t/m^2, one obtains for the longitudinal part of the $\pi^- p \to \gamma^* n$ amplitude (Furlan G. et al. in Ref.2)

$$E_{0+} - 2E_{2-} = \frac{\lambda}{2m_{\pi}^2 f_{\pi}}\sqrt{\frac{(w+m)^2 - m_{\pi}^2}{mw}}\left\{D(t) - \left(1 + \frac{\lambda}{2m}\right)D(m_{\pi}^2 - \lambda^2) + \right. \tag{3}$$

$$\left. + \frac{m_{\pi}^2}{2m}\left[G_A(m_{\pi}^2 - \lambda^2) - \frac{t}{2m}G_P(m_{\pi}^2 - \lambda^2)\right]\right\},$$

where the constant of the $\pi \to \mu + \nu_{\mu}$ decay f_{π} is defined by $\langle 0\,|A_{\mu}(0)|\,\pi(q)\rangle = if_{\pi}q_{\mu}$, $D(t) = -2mG_A(t) + tG_P(t)$, and the quasi-threshold values of the variables are

$$w_{q.thr.} = m + \lambda, \qquad t_{q.thr.} = (m_{\pi}^2 - \lambda^2)\,\frac{m}{m+\lambda}.$$

G_A has been measured in various experiments (first of all, in $\nu n \to \mu^- p$, $\bar{\nu} p \to \mu^+ n$). It is reasonable to use first this result:

$$G_A(t) = G_A(0)(1 - t/M_A^2)^{-2}, \quad G_A(0) = -1.25, \quad M_A = (0.96 \pm 0.03)\ \text{GeV}. \tag{4}$$

However, G_P can be seen to be kinematically suppressed in these experiments in view of its contribution to cross sections to be multiplied by lepton masses (from here, a difficulty of obtaining information on G_P in these experiments). In the μ-capture and β-decay experiments, there is a strong kinematical restriction of the range $|t| \sim 0$—0.01 (GeV/c)^2 in which the weak FF's can be determined, however, with a large error. For example, its measured value for μ capture in hydrogen [12] is $G_P(-0.88m_{\mu}^2) = -8.7 \pm 1.9$. Recently, G_P has been measured in the capture of polarized muons by ^{28}Si nuclei [13].

From formula (3) it is seen that the kinematic suppression of G_P would be absent when the IPE data at the quasi-threshold are used for extracting G_P. On the basis of this method, $G_P(t)$ could be determined in the range up to $t \approx -15m_{\pi}^2$ (which corresponds to

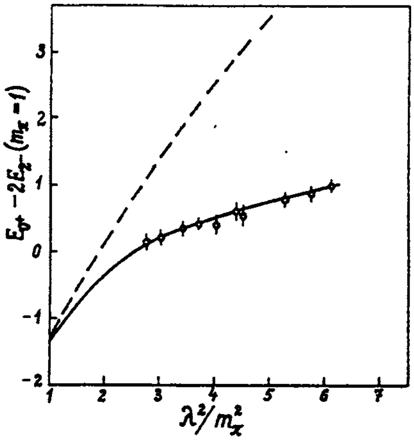

Fig.1. Comparison of calculations in the CA approach to the $\pi^- p \to \gamma^* n$ process for $E_{0+} - 2E_{2-}$ with experimental data: dashed and solid curves correspond to the cases when contribution to G_P is restricted by the pion pole G_P^{π} and taken according to (6), respectively.

$w \approx 1500$ MeV). Due to working at the quasi-threshold, one succeeds in avoiding threshold difficulties which are the case when using the analogous method for analysing electroproduction data.

Further we shall follow the method of work [3]. First, using the $F_1^v(\lambda^2)$ and $F_\pi(\lambda^2)$ values obtained in the analysis of the IPE data on the nucleon [7] we obtain ten points (which can be considered as the experimental ones) for the longitudinal part of the $\pi^- p \to \gamma^* n$ amplitude at the quasi-threshold (Fig.1). For $G_P(t)$ we take the following dispersion relation without subtractions

$$G_P(t) = \frac{2f_\pi g_{\pi N}}{m_\pi^2 - t} + \frac{1}{\pi} \int_{9m_\pi^2}^{\infty} \frac{\rho(t')}{t' - t} dt'. \qquad (5)$$

The residue in the pole $t = m_\pi^2$ is determined by the PCAC relation. In Fig.2, possible contributions to G_P are depicted. When only the π-pole term is considered, it is inconsistent with experimental data (the dashed curve in Fig.1). Since the contributions of nonresonance three-particle states must be suppressed by the phase volume, it is reasonable to approximate the integral in (5) by a pole term. A satisfactory description is obtained if

$$G_P(t) = G_P^{\pi}(t) - \frac{2f_{\pi'} g_{\pi' N}}{m_{\pi'}^2 - t}, \quad 2f_{\pi'} g_{\pi' N} = (1.97 \pm 0.18)\ \text{GeV}, \quad m_{\pi'} = 0.5\ \text{GeV}, \qquad (6)$$

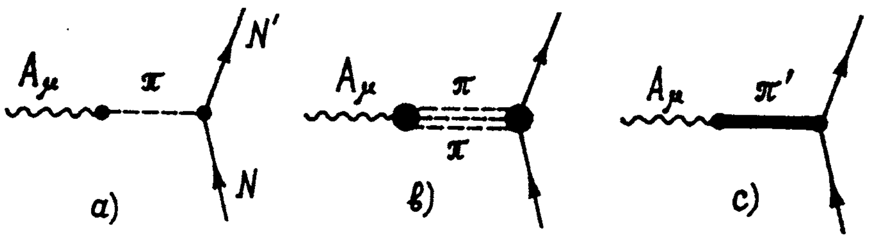

Fig.2. The contributions to G_P of possible intermediate states, coupled with the current A_μ: *a*) one-pion state, *b*) three-pion state, *c*) a resonance with the pion quantum numbers

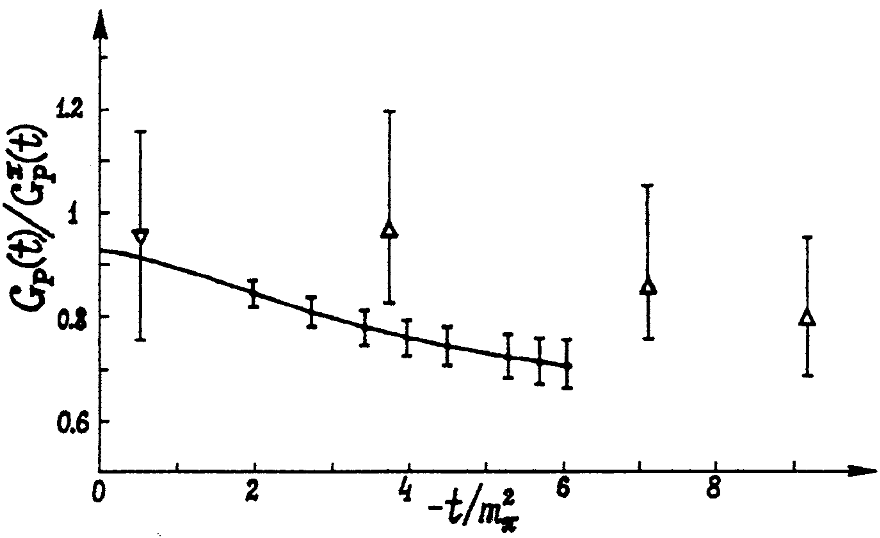

Fig.3. The ratio of $G_P(t)/G_P^{\pi}(t)$. The curve corresponds to formula (6). The points with errors (on the curve) indicate the error corridor for this curve. The results of analysis of data on the μ capture in hydrogen (∇) [12] and on the π^+ electroproduction off the proton near the threshold (Δ) [14] are depicted

where $G_P^{\pi}(t) = 2f_{\pi}g_{\pi N}/(m_{\pi}^2 - t)$, the π' weak-decay constant $f_{\pi'}$ is defined by $\langle 0 | A_{\mu}(0) | \pi'(q') \rangle = if_{\pi'}q'_{\mu}$, $g_{\pi N}(= 13.5)$ and $g_{\pi' N}$ are the coupling constants of the π and π′ states with the nucleon. As it is seen from the definitions of the weak-decay constants, one must expect that $f_{\pi'} << f_{\pi}$, to reflect a tendency of another way (in addition to the Goldstone one) in which the axial current is conserved for vanishing quark masses. That behaviour is demonstrated in various models with some nonlocality which describe chiral symmetry breaking [10,11]. Note that the pole at $t = m_{\pi'}^2$ in Eq.(6), situated considerably lower than the poles of the known contributing states π′(1300) and π′(1770), is highly required for describing the obtained expeirmental data on IPE. In Fig.3, the ratio $G_P(t)/G_P^{\pi}(t)$ is shown. One can see that $G_P(t)$ is determined by this method with a high accuracy. For the comparison, the G_P values, obtained in μ capture in hydrogen [12] and in the recent analysis of data on the π^+ electroproduction off the proton near the threshold [14], are depicted. We see that their results agree with the pion-pole dominance hypothesis in a large range of transfers, unlike our result where this hypothesis is valid in a narrow t range, and outside the range the contribution of continuum is considerable. Note that the contributions of the radial excitations of pion (π′(1300) and π′(1770), which are rather distant from this region, are suppressed, and their account would only slightly increase the mass of π′(500)). The parameters of this pole term in (6) might be changed more considerably if the scalar σ(555) and/or ε(750), discussed at present [15], are confirmed. Then it would be necessary to consider the channel (επ) and a possible multichannel nature of this state. At all events, the conclusion about the necessity of the state in the range 500-800 MeV with $I^G(J^P) = 1^-(0^-)$ for explaining the obtained IPE data will remain valid. Note that recently the state with those parameters has been observed in the $\pi^+\pi^-\pi^-$ system [16] and interpreted as the first radial excitation of pion in the framework of a covariant formalism for two-particle equations used for constructing a relativistic quark model [17]. Note that accepting this designation for π′(500–800) and taking an estimation for the π′ weak-decay constant in the Nambu–Jona-Lasino model, generalized by using effective quark interactions with a finite range, $f_{\pi'} = 0.65$ MeV [10], we obtain $g_{\pi' N} = 1.51$. Of course, for more sure interpretation of π′ the investigation of other processes with π′ is needed, and the presence of this state would raise the question on its SU(3) partners and on careful (re)analyses of the corresponding processes in this energy region.

We see that a subsequent investigation of IPE is necessary for extracting both a unique information about the electromagnetic structure of particles in the sub-$N\bar{N}$ threshold region of the timelike momentum transfers and the nucleon weak structure in the spacelike region. The former is especially interesting now, for example, in connection with the discussed hidden strangeness of the nucleon (e.g., [18]) and quasi-nuclear bound $p\bar{p}$ state [19]. Furthermore, at present, with intense pion beams being available, more detailed experiments are possible aimed also at carrying out a multipole analysis similar to that for photo- and electroproduction (e.g., [20]). That analysis is necessary to study the electromagnetic structure of the nucleon-isobar systems in the timelike momentum-transfer region; for example, in the $P_{33}(1232)$ region it is interesting to verify the λ^2 dependence of the colormagnetic-force contribution found in the constituent quark model [21].

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